An Accurate, Simplified Method for Analyzing Thin Plates Undergoing Large Deflections

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The present analysis examines the behavior of thin, circular, isotropic plates considering geometric nonlinear behavior but not material nonlinearity. The circular plates examined were solid with a fully clamped outer boundary. An approximate solution was determined with the aid of MACSYMA, a symbolic manipulation computer program, using the energy method. The loading cases examined were: a uniform pressure load and a central point load. The solution was based upon a two-term trial function for the plate deflection, with the coefficients written in a new way. This type of trial function allows the dimensionless deflection profile to vary with the loading. This is an improvement over many existing solutions. Results are given for dimensionless plate center deflection and dimensionless radial bending stress at the plate center. Comparisons between the present results and several existing analytical and finite-element solutions are presented. Also, comparisons of the present results and experimental analyses are given, with good agreement.

Nomenclature

a	= plate radius
D	= plate flexural stiffness
E	= Young's modulus
h	= plate thickness
k,k_1,k_2	= general undetermined coefficients
$K=kh$	= dimensionless form of constant k
p	= central point load
P	= dimensionless form of $p\,(=pa^2/Eh^4)$
q	= uniform pressure load
Q	= dimensionless form of $q\,(=qa^4/Eh^4)$
r	= radial position coordinate
S	= dimensionless form of $\sigma_r^b\,(=\sigma_r^b a^2/Eh^2)$
u	= in-plane displacement in the radial direction
V_1,V_2	= strain energies of plate bending and stretching
w	= transverse deflection
$W=w/h$	= dimensionless form of w
W_e	= external work
w_o	= general undetermined coefficient
$W_o=w_o/h$	= dimensionless form of w_o
Δ	= change in quantity
ϵ_r,ϵ_t	= radial and circumferential strains
υ	= Poisson's ratio
Π	= total potential energy
σ_r^b	= radial bending stress

Introduction

ALTHOUGH the roots of classical linear plate theory can be traced back to the early 1800's, only in the last 50 years has significant attention been focused upon the nonlinear behavior of plates.[1] Among the different areas of this theory that have been addressed, geometrically nonlinear behavior has not been resolved adequately. A thin isotropic plate undergoing large deflection is one example of such behavior. Unfortunately, many of the existing analyses of this problem suffer from one of the following shortcomings: 1) the method is so mathematically rigorous that the structures engineer finds it too unwieldy for practical use, 2) the method is relatively easy to apply, but the results obtained are of limited accuracy, or 3) the method has an excessive amount of inherent computations requiring large blocks of computer time and expense.

The objective of the present work is to develop an approximate solution of the problem of thin, circular, isotropic plates undergoing large deflections. It is desired that the solution developed should be simple to use, provide accurate results, and be more efficient computationally than most existing numerical procedures. The present analysis will encompass the following two cases for thin, circular, isotropic plates: 1) fully clamped exterior with uniform pressure loading, and 2) fully clamped exterior with a point load applied at the plate center. By "fully clamped" is meant clamped both flexurally and immovably in the plane. This boundary condition produces the most nonlinear behavior.

Analysis of Uniform Pressure Loading Case

Way[2] developed a solution based upon infinite series expansions, and this work is considered to be an "exact" solution to the problem of a uniformly loaded circular plate undergoing large deflections. Although Way's solution is credited as being exact, it is extremely unwieldy due to the use of infinite series, the procedure for satisfying the boundary conditions, and the excessive amount of intermediate computations required.

Timoshenko[3] proposed an approximate solution, based upon the energy method, which is very simple to apply. This solution is based on the following trial function for plate deflection:

$$w=w_o(1-r^2/a^2)^2 \tag{1}$$

For the case of $\upsilon=0.3$, the constant w_o can be determined from

$$(w_o/h)+0.4835(w_o/h)^3=(3/16)(qa^4/Eh^4)(1-\upsilon^2) \tag{2}$$

Equation (2), as given, was corrected by subsequent investigators[4,5] from its original erroneous form. Although Timoshenko's solution is easy to apply and produces accurate predictions for center deflection, it does not allow for the dimensionless deflection profile (w/w_o) to change with the normalized load. This leads to incorrect predictions for

Received Jan. 30, 1987; presented as Paper 87-0808 at the AIAA/ASME/AHS/ASEE 28th Structures, Structural Dynamics and Materials Conference, Monterey, CA, April 6-8, 1987; revision received June 28, 1987.

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the dimensionless deflection at intermediate points on the plate between the plate center and the boundary. Thus, the stresses predicted by Timoshenko's theory are considerably higher than Way's as will be shown.

Nádai[6] proposed an approximate method based on an assumed expression for the plate slope

$$\frac{\mathrm{d}w}{\mathrm{d}r} = C\left[\left(\frac{r}{a}\right) - \left(\frac{r}{a}\right)^n\right] \quad (3)$$

The constants C and n were selected in a manner so as to make the pressure (q) as near a constant value as possible. For the case of $\upsilon = 0.25$, Nádai proposed the following relation for the central deflection:

$$(w_o/h) + 0.583(w_o/h)^3 = (3/16)(qa^4/Eh^4)(1-\upsilon^2) \quad (4)$$

Although Nádai's result is simple to apply, his method has several shortcomings. Primarily, Nádai's method does not allow for the dimensionless deflection profile to change with the dimensionless loading. Also, the procedure for determining C is very labor-intensive, requiring a trial-and-error method for determining the "best" value for C. Nádai's final choices for the C values resulted in poor approximations to uniform loading, with variations as high as 17%. Furthermore, the coefficients in Eq. (4) for (w_o/h) were determined from curve-fitting techniques using only three data points.

The present work is based upon the energy method. A two-term trial function is used for predicting out-of-plane deflections. The constants for the trial function have been written in a new way. The coefficient of the second term has been written as the product of two undetermined constants. By using this two-term trial function, the dimensionless deflection profile can change with the applied loading. Consequently, other quantities of interest derived from the deflection will be predicted more accurately. Various previous investigators[7-10] have noted that the dimensionless deflection profile (DDP) of a plate undergoing large deflections changes as the applied loading is increased. Physically, this is explained by noting that at very small loadings, the DDP coincides with that predicted by small-deflection (linear) plate theory, but, as the loading is increased, membrane action is developed. Thus, at the limit of exceedingly large loading, the DDP approaches that of a pure membrane.

Most of the detailed analytical manipulations in the present analysis were performed using the MACSYMA symbolic manipulation computer program. This program allows for very tedious analytical calculations to be completed in a fraction of the time required to perform them by hand, with chances for algebraic errors virtually eliminated.

For the case of thin, circular plates of isotropic material, the following expressions were derived in classical plate theory (cf. Timoshenko and Woinowsky-Krieger[11]). For the axisymmetric case, the strain energy due to bending is

$$V_1 = \pi D \int_0^a \left[\left(\frac{\mathrm{d}^2 w}{\mathrm{d}r^2}\right)^2 + \left(\frac{1}{r^2}\right)\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right)^2 + \left(\frac{2\upsilon}{r}\right)\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right)\left(\frac{\mathrm{d}^2 w}{\mathrm{d}r^2}\right)\right] r\,\mathrm{d}r \quad (5)$$

where D is the plate flexural rigidity. The strain energy due to stretching of the middle plane is

$$V_2 = \left[\frac{\pi E h}{1-\upsilon^2}\right] \int_0^a (\epsilon_r^2 + \epsilon_t^2 + 2\upsilon\epsilon_r\epsilon_t) r\,\mathrm{d}r \quad (6)$$

where the radial and circumferential midplane strains are

$$\epsilon_r = \left(\frac{\mathrm{d}u}{\mathrm{d}r}\right) + \left(\frac{1}{2}\right)\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right)^2 \quad (7)$$

$$\epsilon_t = \frac{u}{r} \quad (8)$$

The plate deflection is assumed as follows:

$$w = w_o(1 - r^2/a^2)^2 + k w_o^2 (1 - r^2/a^2)^3 \quad (9)$$

where w_o and k are general undetermined coefficients. The boundary conditions for the clamped edge are

$$w(a) = 0, \qquad w_{,r}(a) = 0 \quad (10)$$

where the comma denotes partial differentiation with respect to the subscript. Also, two regularity conditions at the plate center apply

$$w(0) \text{ finite}, \qquad w_{,r}(0) = 0 \quad (11)$$

It can be seen that Eq. (9) satisfies conditions (10) and (11).

A radial displacement function is assumed as

$$u = r(a - r)(k_1 + k_2 r) \quad (12)$$

where k_1 and k_2 are two general undetermined coefficients. This function is subject to the following boundary and regularity conditions:

$$u(a) = 0, \qquad u(0) = 0 \quad (13)$$

By substituting the appropriate derivatives of Eq. (9) into Eq. (5) and evaluating the integral, one can determine V_1. By substituting Eqs. (9) and (12) into Eqs. (7) and (8), the expressions for strain can be determined. The quantity V_2 can be found by use of Eqs. (9), (12), (7), and (8) in Eq. (6) and evaluating the integral. The external work applied to the plate can be determined from

$$W_e = \int_0^{2\pi} \int_0^a q w(r) r\,\mathrm{d}r\,\mathrm{d}\theta \quad (14)$$

The total potential energy of the plate can be expressed as

$$\Pi = V_1 + V_2 - W_e \quad (15)$$

In order for the plate to be in equilibrium, the potential energy must be a minimum. Therefore, the coefficients k_1 and k_2 must be assigned so that they minimize the expression for the total potential energy. Thus, k_1 and k_2 may be determined from standard differential calculus methods.

To determine w_o and k, the theory of virtual work is invoked

$$\delta W_e = \delta(V_1 + V_2) \quad (16)$$

where δ denotes a change in the total quantity due to a small virtual displacement. The variation in the total work is found from

$$\delta W_e = 2\pi \int_0^a q(\delta w) r\,\mathrm{d}r \quad (17)$$

In Eq. (9), w is given as a function of w_o and k; thus

$$\delta w = \left(\frac{\partial w}{\partial w_o}\right)\delta w_o + \left(\frac{\partial w}{\partial k}\right)\delta k \quad (18)$$

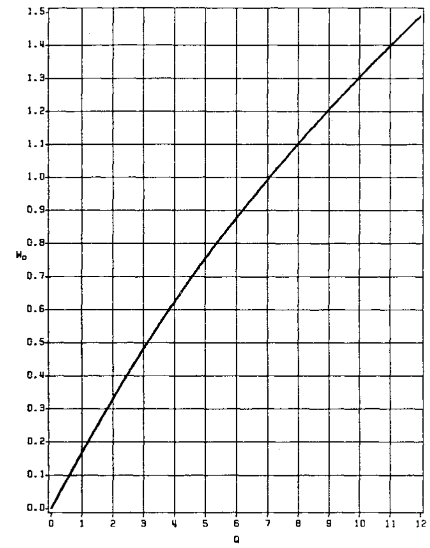

Fig. 1 Coefficient W_o vs dimensionless pressure load ($\upsilon=0.3$).

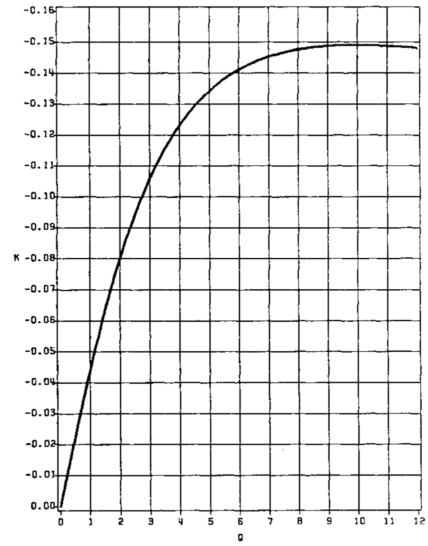

Fig. 2 Coefficient K vs dimensionless pressure load ($\upsilon=0.3$).

Similarly,

$$\delta(V_1+V_2)=\left[\frac{\partial(V_1+V_2)}{\partial w_o}\right]\delta w_o+\left[\frac{\partial(V_1+V_2)}{\partial k}\right]\delta k \quad (19)$$

Substituting Eqs. (17-19) into Eq. (16) gives

$$\left[2\pi q\int_0^a\left(\frac{\partial w}{\partial w_o}\right)r\,\mathrm{d}r\right]\delta w_o+\left[2\pi q\int_0^a\left(\frac{\partial w}{\partial k}\right)r\,\mathrm{d}r\right]\delta k$$
$$=\left[\frac{\partial(V_1+V_2)}{\partial w_o}\right]\delta w_o+\left[\frac{\partial(V_1+V_2)}{\partial k}\right]\delta k \quad (20)$$

Equating similar terms in Eq. (20) gives

$$2\pi q\int_0^a\left(\frac{\partial w}{\partial w_o}\right)r\,\mathrm{d}r=\frac{\partial(V_1+V_2)}{\partial w_o} \quad (21a)$$

$$2\pi q\int_0^a\left(\frac{\partial w}{\partial k}\right)r\,\mathrm{d}r=\frac{\partial(V_1+V_2)}{\partial k} \quad (21b)$$

Evaluating the derivatives and integrals in Eqs. (21) results in

$$f_1(W_o,K;\,\upsilon,Q)=0 \quad (22a)$$

$$f_2(W_o,K;\,\upsilon,Q)=0 \quad (22b)$$

that is a system of two coupled nonlinear algebraic equations in W_o and K, with parameters υ and Q. The quantities W_o, K, and Q are dimensionless forms of w_o, k, and q.

Due to the complicated forms of Eqs. (22), it was necessary to specify a value for υ to simplify the equations. For the case when $\upsilon=0.3$, Eqs. (22) (with coefficients rounded to two decimal places) reduce to

$$3QKW_o+2Q=12.45K^4W_o^7+39.37K^3W_o^6+47.89K^2W_o^5$$
$$+26.56KW_o^4+21.10K^2W_o^3+5.67W_o^3$$
$$+26.37KW_o^2+11.72W_o \quad (23a)$$

$$Q=4.15K^3W_o^6+11.25K^2W_o^5+10.64KW_o^4+3.54W_o^3$$
$$+7.03KW_o^2+5.86W_o \quad (23b)$$

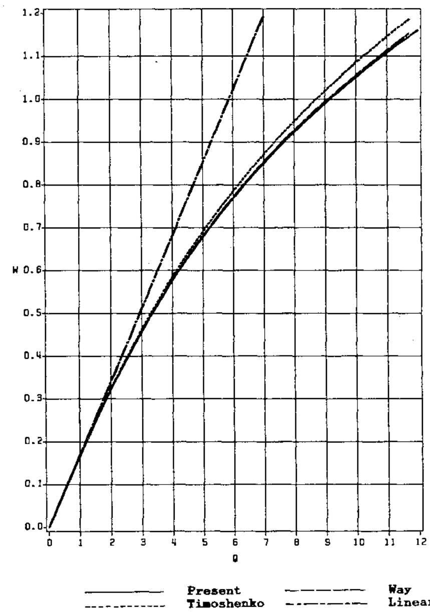

Fig. 3 Dimensionless center deflection vs dimensionless pressure load ($\upsilon=0.3$).

Equations (23) were solved numerically for W_o and K using Gauss-Seidel iteration and specifying a value for Q. The results are shown graphically in Figs. 1 and 2. It can be seen in Fig. 1 that W_o varies with Q in a slightly nonlinear fashion, while Fig. 2 shows highly nonlinear behavior.

Numerical Results for Uniform Pressure Loading Case

Results are shown in Fig. 3 for the normalized plate center deflection vs the dimensionless load for a 0.3 value of Poisson's ratio. In addition to the present results, the predictions of Timoshenko, Way, and linear theory are shown. Here, the plot of the present solution is almost coincident with Way's prediction.[2] It is noted that significant deviations from linear theory begin to ccur at deflections greater than half of the plate thickness. The present results are in excellent agreement with Way's predictions and only at higher loadings can the two results be distinguished from one another. Timoshenko's analysis predicts less plate stiffness than both the present one and Way's, but the predictions are

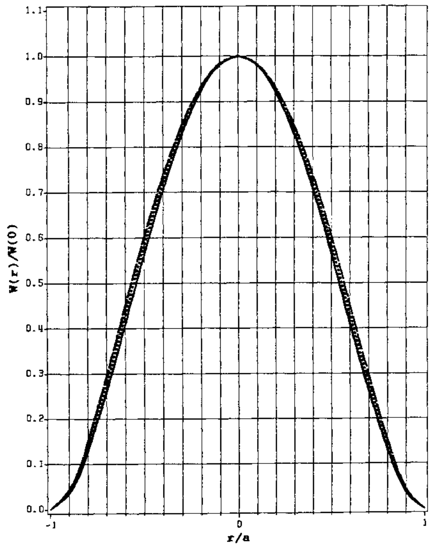

Fig. 4 Dimensionless deflection distribution ($\upsilon = 0.3$). The lower, middle, and upper curves correspond to $Q = 0$, 6, and 12, respectively.

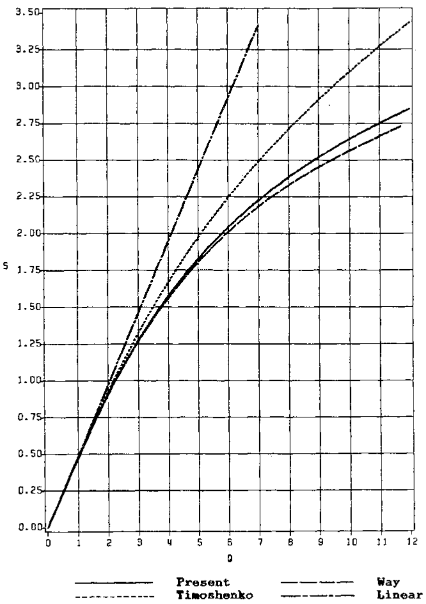

Fig. 5 Dimensionless radial bending stress at center vs dimensionless pressure load ($\upsilon = 0.3$).

Similar results were obtained for the case of $\upsilon = 0.25$, with Nádai's results substituted for Way's.[12] Way made a comparison between his results and Nádai's, in which the latter solution was found to slightly under predict the center deflections. The present method predicts dimensionless center deflections slightly larger than Nádai's, and Timoshenko's analysis produces reasonably accurate predictions for the plate center deflection.

The dimensionless deflection profile is shown in Fig. 4 for $\upsilon = 0.3$ and dimensionless loadings 0, 6, and 12. The case of dimensionless loading of 0 corresponds to classical linear theory. It can be seen that at higher loads, the deflection profile "bows out" more. In the present method, by expressing the plate deflection as a two-term trial function with loading dependent coefficients, one can incorporate loading effects into the dimensionless deflection distribution. This allows for a more accurate prediction of the deflection distribution than theories without this characteristic.

The normalized radial bending stresses at the plate center are plotted in Fig. 5 against the normalized loading for the case of a 0.3 value for υ. The present results are shown along with the predictions of Timoshenko, Way, and linear theory. Unlike the case for deflections, Timoshenko's dimensionless stress predictions are considerably different from those of Way. This was expected since the bending stresses involve derivatives of the deflections. It can be seen that the results of the current method are much closer to those of Way than Timoshenko's. Similar results were also found for a 0.25 value of Poisson's ratio.[12]

Way,[2] in 1934, conducted experiments for comparison with his analytical results. As in the case of Way's analysis, the present method gives lower deflections than those determined experimentally. The discrepancies may be attributed to two sources: 1) slipping and rotating at the boundary (i.e., imperfect clamped boundary conditions), and 2) inaccuracy in measuring deflection.

McPherson, Ramberg, and Levy[13] performed an extensive experimental analysis of the uniformly loaded circular plate problem. Figure 6 shows some selected results from their investigations along with the present predictions. The results from Ref. 13 have been normalized to compare their work with the present predictions. The figure shows the results of three plates denoted as plates E, F, and L with values of 0.29, 0.29, and 0.31 for Poisson's ratio, respectively. The present results (for $\upsilon = 0.3$) are also shown. The analytical

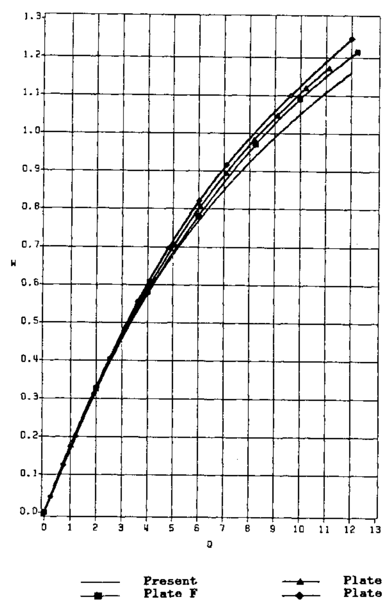

Fig. 6 Dimensionless center deflection vs dimensionless pressure load.

predictions agree well with the experimental deflections but are slightly lower. A part of the deviation of plates E and F may be attributed to their value of Poisson's ratio being less than 0.3; Poisson's ratio has the effect on the present theory of shifting the deflection curve up for lower values of υ and down for higher values of υ. A more significant source of error is experimental difficulty of achieving perfect clamping at the edge.

Although the finite-element method (FEM) has increased in popularity several-fold in recent years, the present investigators were unable to find a thorough FEM analysis of the solid circular plate with uniform loading and geometric nonlinearity. Krayterman and Fu[14] recently examined this problem using the integral collocation method and prepared a finite-element model using ANSYS (Swanson Analysis Systems, Inc.) to compare with their results. Table 1 shows the results of their FEM model along with the predictions of the present method. The calculations were conducted using

Table 1 Comparison of results from the present method and the finite-element method

Method	*Q*	*w*, in. (center)	% Δ	σ_r^b, ksi (center)	% Δ
Present	5.517	0.36359	— —	1.410	— —
FEM 10 element		0.36512	−0.42	1.402	−0.57
FEM 20 element		0.36442	−0.23	1.393	−1.21
Present	55.172	1.12970	— —	3.120	— —
FEM 10 element		1.15098	−1.88	3.042	2.50
FEM 20 element		1.14565	−1.41	3.010	3.53
Present	114.670	1.47216	— —	3.566	— —
FEM 10 element		1.51175	−2.69	3.648	−2.30
FEM 20 element		1.50376	−2.15	3.585	−0.53

$\upsilon = 0.3$, thickness $h = 0.5$ in., radius $a = 100$ in., and Young's modulus $E = 29{,}000$ ksi. Although there were little FEM data for comparison, it may be seen that there is good agreement. The maximum difference between the present analysis and the twenty-element solution for center deflection is 2.15%. The maximum difference between the radial bending stress at the plate center for the present method and the twenty-element solution is 3.53%.

Stein[15] analyzed the present problem using an incremental method, and recently Jang et al.[10] analyzed it using the method of differential quadrature. In both instances, excellent agreement was obtained with Way's exact solution; therefore, in the interest of brevity, no comparisons are presented here.

Analysis of Central Point Loading Case

Timoshenko and Woinowsky-Krieger[11] derived an approximate solution using the Galerkin method. The deflection trial function used was

$$w = w_o[1 - (r^2/a^2) + 2(r^2/a^2)\ell n(r/a)] \tag{24}$$

This expression has the same dimensionless deflection distribution (w/w_o) as the linear solution. For the case of $\upsilon = 0.3$, the constant w_o is found from

$$(w_o/h) + 0.443(w_o/h)^3 = 0.217pa^2/(Eh^4) \tag{25}$$

Although this solution is easy to use, it does not permit the dimensionless deflection profile to change with the applied loading.

Banerjee[16] has approximately solved this problem using the energy method with a modified expression for the total strain energy. The same trial function was used as Eq. (24) above as well as the principle of minimum potential energy to obtain (for $\upsilon = 0.3$) an expression identical to Eq. (25) with 0.443 replaced by 0.43.

Chien and Yeh[17] found an approximate solution using the perturbation method. Although their results appear to be quite consistent with other solutions in the literature, the method is extremely laborious. Although the first terms of their proposed series are relatively easy to find, for each additional term desired in the series, the amount of additional work grows substantially. The results of Chien and Yeh are limited in that they are expressed only for specific points on the plate, namely, at the center and the edge. No general results were introduced for determining quantities at intermediate points on the plate.

Schmidt[18] also used the perturbation method to derive an approximate solution for this problem. Schmidt's procedure is similar to that of Chien and Yeh, and thus, his methodology and results are subject to the same criticisms raised above.

The procedure for solving the central point load problem is identical to the procedure used in the uniform pressure problem, with a few minor variations. The expressions for the bending and the stretching strain energies are identical to those stated in Eqs. (5) and (6). The assumed transverse deflection trial function is

$$w = w_o[1 - (r^2/a^2) + 2(r^2/a^2)\ell n(r/a)] + kw_o^2[1 - r^2/a^2]^2 \tag{26}$$

This trial function is subject to the same regularity and boundary conditions as in Eqs. (10) and (11). The radial displacement trial function is given by Eq. (12).

The bending and stretching strain energies are found in the same manner as outlined before in the uniform pressure loading case. Since the load is applied at the plate center, the external work is

$$W_e = pw(0) \tag{27}$$

Applying the principle of minimum potential energy and the principle of virtual work, as discussed in the uniform pressure case, finally leads to a system of two coupled nonlinear algebraic equations of the form

$$g_1(W_o, K;\ \upsilon, P) = 0$$
$$g_2(W_o, K;\ \upsilon, P) = 0 \tag{28}$$

where W_o and K are dimensionless forms for w_o and k, and P is a dimensionless form of p.

Again, due to the complicated forms of Eqs. (28), it was necessary to assign a value of υ and solve the system of equations numerically. For the case of $\upsilon = 0.3$, Eqs. (28) (with coefficients rounded to two decimal places) reduce to

$$2PKW_o + P = 5.93\,K^4W_o^7 + 17.71\,K^3W_o^6 + 20.45\,K^2W_o^5 + 10.77\,KW_o^4 + 12.27\,K^2W_o^3 + 2.16\,W_o^3 + 13.81\,KW_o^2 + 4.60\,W_o \tag{29a}$$

$$P = 2.97\,K^3W_o^6 + 7.59\,K^2W_o^5 + 6.82\,KW_o^4 + 2.15\,W_o^3 + 6.14\,KW_o^2 + 4.60\,W_o \tag{29b}$$

Plots of W_o and K vs P are shown in Figs. 7 and 8. Both W_o and K are nonlinear in P, but K is positive for this loading case.

Numerical Results for Central Point Loading Case

The present results for the normalized plate center deflections vs normalized loading for $\upsilon = 0.3$ are shown in Fig. 9. Also shown are the results from linear theory, Banerjee, and Chien and Yeh. As in the uniform pressure loading case, significant deviations from linear theory begin at deflections

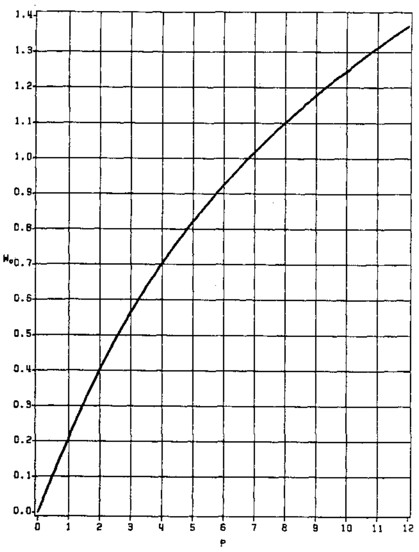

Fig. 7 Coefficient W_o vs dimensionless concentrated load ($\upsilon = 0.3$).

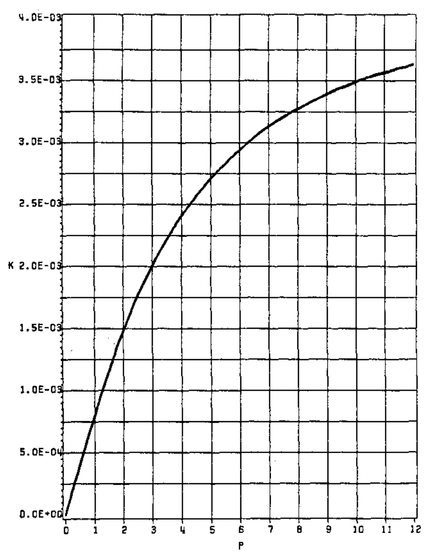

Fig. 8 Coefficient K vs dimensionless concentrated load ($\upsilon = 0.3$).

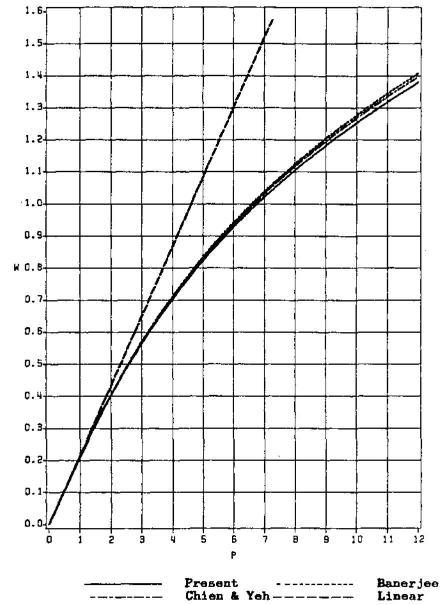

Fig. 9 Dimensionless center deflection vs dimensionless concentrated load ($\upsilon = 0.3$).

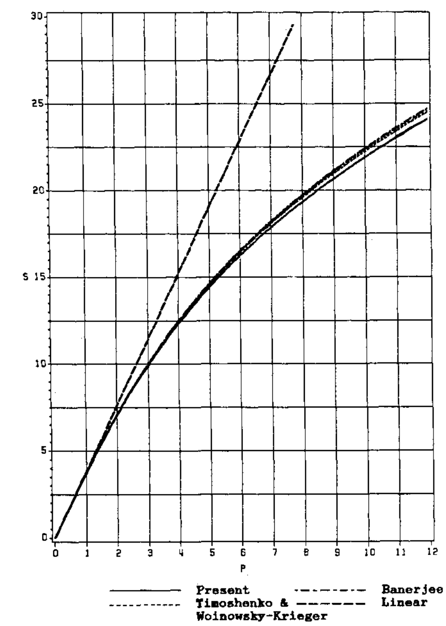

Fig. 10 Dimensionless radial bending stress at center vs dimensionless concentrated load ($\upsilon = 0.3$).

equal to half the plate thickness. For the range of loading considered, results of the present method are consistent with all the other nonlinear solutions shown as well as those of Refs. 11 and 18 (not shown for brevity).[12]

Determining the radial bending stress at the plate center due to a concentrated load creates difficulties for many theories. The theory of elasticity predicts infinite stresses at the origin for a point load applied at the origin of a three-dimensional semi-infinite medium, due to a mathematical singularity. Although plate theory is not as precise as the theory of elasticity, most solutions based on large deflection theory will also predict infinite radial bending stresses at the plate center. A common procedure to avoid the infinite stress predictions is to replace the point load by a distributed load about the circumference of an infinitesimal circle located at the plate center. The stress is then calculated at the edge of the small circle instead of at the plate center, thus, eliminating the mathematical singularity. This procedure has been followed in the present work.

In Fig. 10 the radial bending stress is plotted as a function of the normalized loading. Shown in the figure are results from the present method, linear theory, and Refs. 11 and 16 for the case of 0.3 Poisson's ratio. The central circle, over which the point load is assumed to be distributed, is arbitrarily selected as having a radius of 1/1000 of the plate radius. It may be seen that the current method is consistent with the previous theories. At the dimensionless loading of $P = 12$, the difference between the present method and Banerjee's results is less than 2.5%. Neither Ref. 17 nor Ref. 18 provided results for the radial bending stress at the plate center. Since they did not give general expressions for the deflection distribution, it is difficult for subsequent investigators to use their results in the classical bending moment and stress equations.

Unlike the uniform pressure loading case, there are relatively little experimental data available in the literature on the central point load problem. Foulkes and Onat[19] made an extensive experimental investigation of thin isotropic circular plates subjected to central point loads. Their investigation was aimed primarily at the plastic-range behavior of plates. It was hoped that the results from their work in the elastic region might correlate with the present study. Unfortunately, few of those test cases can be considered as "thin" plates. Those that qualify do not correlate well with established theories in the elastic region. Their experimental data predicted plate stiffnesses lower than linear theory, which indicates either very poor clamping at the plate edge or localized yielding at the plate center. Unfortunately, the present investigators were unable to locate any other experimental investigations on homogeneous isotropic plates subjected to a central point load.

Summary and Conclusions

For the two loading cases examined, the present method has proven very accurate in analyzing the behavior of thin, circular, isotropic plates undergoing large deflections. Where exact results were available for plate center deflections, the present method predicted results very close to the exact solution. Where only approximate solutions were available, the present method produced results consitent with previous analyses. The present solution has also produced results consistent with previous experimental and finite-element analyses.

The present solution is expressed as a two-term trial function for the plate deflection. The solution, expressed in this form, can easily be used for practical structural analysis. Also, it is straightforward to determine other quantities of interest (e.g., stress, plate curvature, bending moment) from the deflection expression. An improvement of the present method over many existing analyses is that it allows the dimensionless deflection profile to vary with the applied loading. This has been accomplished by expressing deflection as a two-term trial function with coefficients that change with the applied loading. This attribute allows the current method to predict more accurately the plate deflection and other quantities at intermediate points between the plate center and the plate boundary. Since a system of only two equations needs to be solved simultaneously, the present method shoud be more efficient computationally that most existing numerical methods.

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